

# Lecture 4

Introduction to spatial modelling – spatial data types

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- Why spatial modelling?
- Why is spatial modelling computationally expensive?
- Different data types:
  - Modelling in discrete space – areal data
  - Modelling in continuous space – geo-referenced data
  - Modelling continuous space – spatial point process data
- representation of data structures in R

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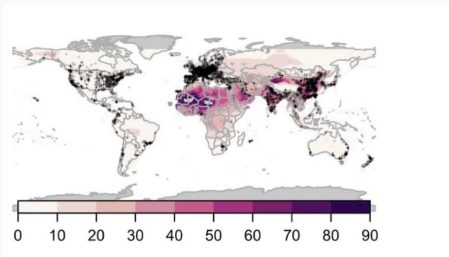
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We will see that `inlabru` can help...

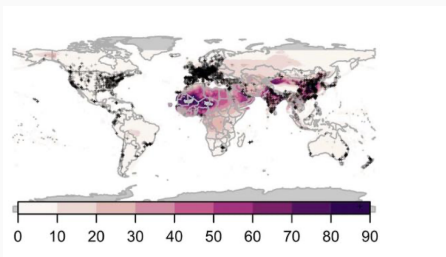
## Example: Global PM 2.5



Exposure to air pollution; particulate matter  $< 2.5\mu\text{m}$  (PM 2.5)

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**spatial dependence + complex observation processes**

## Spatial modelling: why necessary?

- standard models assume independent observations
- spatial data are (typically) not independent
- two nearby observations are similar → not providing independent information

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There are many different ways of “being dependent”, hence these **components** and their properties vary with each application.

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- INLA (and hence `inlabru`) much faster...

aims vary among different studies, e.g.:

- describing spatial patterns and structures
- understanding spatial patterns and structures
- linking data observed in space to covariates while accounting for spatial autocorrelation
- predicting into unobserved locations

## Spatial modelling: aims

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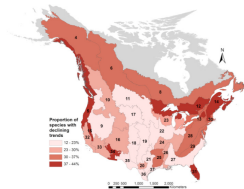
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aims also vary with different types of spatial data...

# Types of spatial data

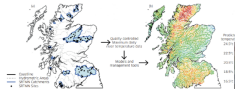
We can distinguish three types of spatial data structures

## Areal data



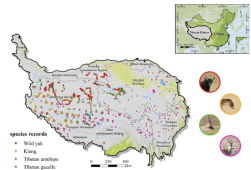
Map of bird conservation regions (BCRs) showing the proportion of bird species within each region showing a declining trend

## Geostatistical data



Scotland river temperature monitoring network

## Point-referenced data



Occurrence records of four ungulate species in the Tibet,

## **Discrete space:**

- Data on a spatial grid (areal data)

## **Continuous space:**

- Geostatistical (geo-referenced) data
- Spatial point patterns (point-referenced data)

Model components are used to reflect spatial dependence structures in discrete and continuous space.

## Discrete space: areal data

- Data on a (regular or irregular) spatial grid
- Examples: number of individuals in a region, average rainfall in a province
- Originally point or geostatistical data; gridded for practical reasons

**Observed response(s):** Measurement associated with each grid cell or areal unit

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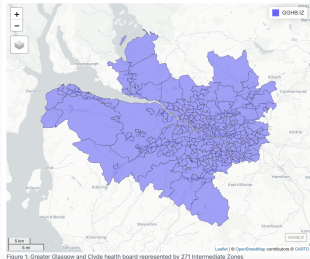
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**here:** spatial structure represented by Gauss Markov random field (GMRF)

## respiratory hospitalisations in Glasgow:

- areal data on respiratory hospitalisations
- 271 Intermediate Zones (IZ) in Greater Glasgow and Clyde health board
- covariates concern air pollution concentration and socio-economic deprivation in Scotland



## Standardized Mortality Ratios (SMR)

Response here:

disease risk, estimated using **Standardized Mortality Ratios (SMR)**: for spatial areal unit  $i$  SMR is the ratio between the observed ( $Y_i$ ) and expected ( $E_i$ ) number of cases:

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$SMR > 1$ : more observed cases than expected

↪ **high risk area**  $SMR < 1$ , fewer observed cases than expected

↪ **low risk area**

## Continuous space: geostatistical data

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- measured at a finite set of locations
- examples: nutrient levels in soil, salinity in the sea

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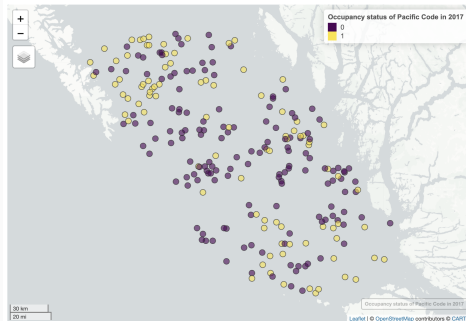
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**here:** spatial structure represented by Gaussian random field; approximated by a continuously indexed Gauss Markov random field

# the data we will use

- Pacific Cod (*Gadus macrocephalus*)
- trawl survey in Queen Charlotte Sound



## Continuous space: spatial point patterns

- Locations of objects/events in space (typically 2D)
- Examples: tree locations, animal groups, earthquakes

**Observed response(s):**  $x,y$  coordinates (sometimes also additional measurements,; "marks")

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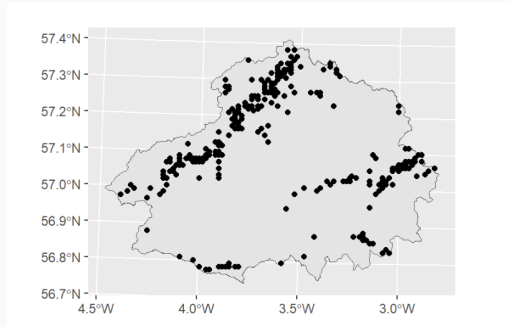
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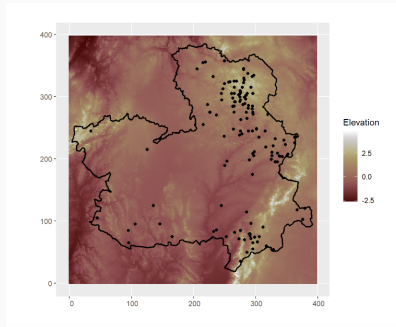
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location of sightings of the ringlet butterfly Scotland's Cairngorms National Park



# the data we will use

locations of forest fires in the Castilla-La Mancha region of Spain between 1998 and 2007



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    ↪ the SPDE approach
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  - different types of models can be efficiently fitted with INLA/inlabru
- ↪ within the same framework
  - different data structures can be jointly modelled

## Point patterns:

- Data format: x,y coordinates
- Optional: marks
- Aim: modelling pattern formed by locations; locations are random

## Geostatistical data:

- Data format: x,y coordinates
- Measurements mandatory
- Aim: model continuous process based on measurements; locations of measurements are not random/have been chosen

## take-home message

- all spatial models discussed as part of this course are special cases of latent Gaussian models
- different spatial terms are needed for different spatial data structures
- the SPDE approach unifies approximation as all data structures are spatially referenced in continuous space
- `inlabru` efficiently fits these models

**Practical 4:** introduction to spatial data wrangling and manipulation

- exploring tools for visualization and wrangling
- for different spatial data structures

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- replaces `sp`

terra: designed for working with spatial **raster** and **vector** data

## Key Features

- handles large raster datasets efficiently
- provides a unified framework for raster and vector data structures
- supports common GIS operations:
  - reprojection, resampling, cropping, masking
  - raster and map algebra
  - zonal statistics, extraction, distance calculations
- integrates with `sf` for combined raster–vector workflows
- supports numerous spatial data formats via the GDAL library.
- successor to `raster` package; faster, more memory-efficient tools

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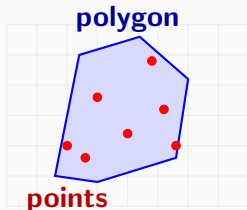
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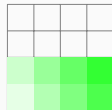
#### sf: vector data

- discrete geometric objects: points, lines, polygons
- store exact shapes and boundaries
- used for administrative areas, roads, sample sites



#### terra: raster data

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- each cell stores a numeric value (e.g., elevation, temperature)
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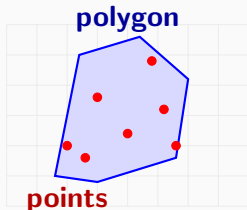


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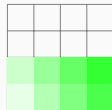
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further slides

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- introduced in 2016, built on the Simple Features standard
- uses tidyverse-friendly data frame structure
- geometry stored as a list-column (sfc)
- works seamlessly with dplyr, ggplot2, tidyr
- automatic CRS handling and conversions

sf is a modern replacement of the sp package

## old: sp

- introduced in early 2000s
- complex S4 class system
- separate objects for geometry and attributes
- limited integration with dplyr/ggplot2
- manual handling of coordinate reference systems

## modern: sf

- introduced in 2016, built on the Simple Features standard
- uses tidyverse-friendly data frame structure
- geometry stored as a list-column (sfc)
- works seamlessly with dplyr, ggplot2, tidyr
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sf simplifies workflows, speeds up computation, and integrates